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TURBO
Continuous change

اسئلة مقترحة :

معادلات تفاضلية عادية 1

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Continuous change

مادة
الفيرست

السؤال الاول :

An integrating factor of the following linear D.E((e^-2√ x/ √ x)-(y/√ x) x' =1

Integrating factor for D.E $\left(\frac{e^{-3\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)x' = 1$

$$\boxed{y' + P(x)y = Q(x)}$$

Standered form. نشكلها على ال

$$* \left(\frac{e^{-3\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1 \Rightarrow \left(\frac{e^{-3\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)dx = dy$$

$$y' = \frac{e^{-3\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \Rightarrow \boxed{y' + \frac{y}{\sqrt{x}} = \frac{e^{-3\sqrt{x}}}{\sqrt{x}}}$$

$$\mu(x) = e^{\int P(x)\,dx} \Rightarrow \int \frac{1}{x^{\frac{1}{2}}} = \int x^{-\frac{1}{2}} = \boxed{2x^{\frac{1}{2}}}$$

$$\boxed{\mu(x) = e^{2\sqrt{x}}}$$

السؤال الثاني :

the order of the D.E $x=((dy/dx)+(dy/dx)^2+(dy/dx)^3...+(dy/dx)^{100})*(dy/dx)$ is equal

السؤال الثالث:

A solution for the I.V.P (dy/dx)=(1/(e^y)-x) ,y(1)=0 is.

السؤال الرابع:

the D.E whose general solution is cos(y-x)=e^-x , (where c is arbitrary constant) is

السؤال الخامس:

the largest interval for which the I.V.P : Y'+(Y/Sin(2x)ln3$\sqrt{\ }$(3-1)^2)=x ,y(-1/2)=3 has a uniqe solution is

السؤال السادس:

Determine a function M(x,y) ,so that the D.E ,
M(x,y)dx +(x lnx+(lnx/y))dy=0 , x , y>0

D.E is exact.

$M(x,y)\,dx + \left(x\ln y + \frac{\ln x}{y}\right)dy = 0$, find $M(x,y)$.

$M_y = N_x = \ln y + \frac{1}{xy}$

$\int M_y = \int \boxed{\int \ln y} + \frac{1}{xy}\,dy$

تكامل اجزاء .

$\boxed{M(x,y) = y\ln y - y + \frac{\ln y}{x} + g(x)}$

$* \int \ln y\,dy$

$u = \ln y \qquad dv = dy$

$du = \frac{1}{y} \qquad v = y$

$uv - \int v\,du$

$y\ln y - \int y \times \frac{1}{y}$

$\boxed{y\ln y - y}$

السؤال السابع:

if N(x,y)-M(x,y) =2y^2 M(x,y),then the solution to the D.E ,((M(x,y)/Secx)dx)-(N(x,y)/y)dy)=0 ,y>0 is

$x y' = y(\ln x + \ln y)$

$\to x y' = y \ln xy$

$V = xy \Rightarrow \boxed{V' = x y' + y} \qquad \boxed{y = \frac{V}{x}}$

$\to V' - y = y \ln V \Rightarrow V' = y\ln V + y \Rightarrow V' = y(\ln V + 1)$

$\frac{dV}{dx} = \frac{V}{x}(\ln V + 1) \Rightarrow \int \frac{dx}{x} = \int \frac{dv}{v(\ln v + 1)}$

$\ln|x| + c = \int \frac{1}{v(\ln v + 1)}\,dv$

$u = \ln v$, $du = \frac{dv}{v}$ $\Rightarrow \int \frac{\cancel{v}\,du}{\cancel{v}(u+1)} = \ln|u+1| = \ln|x| + c$

$\ln|\ln v + 1| = \ln|x| + c$

$= \boxed{\ln|\ln xy + 1| = \ln|x| + c}$

السؤال الثامن:

Solve the D.E xy'=y(ln3√ y-ln3√ x) , x,y>0

Solve : $xy' = y(\ln\sqrt[3]{y} - \ln\sqrt[3]{x})$, $x, y > 0$

$$y' = \frac{y}{x}\left(\ln\sqrt[3]{\frac{y}{x}}\right) \Rightarrow y' = \frac{y}{x}\left(\frac{1}{3}\ln\frac{y}{x}\right)$$

*Homogenous.

$$\boxed{v = \frac{y}{x}} \Rightarrow \boxed{y = v \times x} \Rightarrow v + xv' = y'$$

$$v + xv' = v\left(\frac{1}{3}\ln v\right) \Rightarrow xv' = v\left(\frac{1}{3}\ln v - 1\right)$$

$$x\frac{dv}{dx} = v\left(\frac{1}{3}\ln v - 1\right) \Rightarrow \int \frac{dv}{v(\frac{1}{3}\ln v - 1)} = \int \frac{dx}{x}$$

$u = \ln v$

$$du = \frac{1}{v}dv \Rightarrow \boxed{dv = du\, v}$$

بالتعويض:

$$\Rightarrow \int \frac{du\, \cancel{v}}{\cancel{v}(\frac{1}{3}u - 1)} = 3\ln(u-1) = \boxed{\ln(u-1)^3}$$

$$\boxed{\ln\left(\ln\frac{y}{x} - 1\right)^3 = \ln x}$$ #

السؤال التاسع:

Solve the D.E (dy/dx)+x+y+1=((x+y)^2)*(e^3x)

Solve:-

$$\frac{dy}{dx} + x + y + 1 = (x+y)^2 e^{3x}$$

$$\boxed{u = x + y} \Rightarrow y' + u + 1 = (u)^2 e^{3x}$$

$$\boxed{u' = 1 + y'}$$

$$\boxed{y' = u' - 1} \Rightarrow u' + u = u^2 e^{3x}$$

$\Rightarrow$ Bernolli

$P(x) = 1$, $Q(x) = e^{3x}$, $n = 2$

$$\boxed{v = u^{1-n}} \Rightarrow \boxed{v = u^{-1}}$$

$$v' + (1-n)p(x)\,v = (1-n)Q(x)$$

$$v' + (-1)\times(1)\times v = (-1)(e^{3x}) \Rightarrow v' - v = -e^{3x} \quad \text{linear in } v$$

$$P(x) = -1 \qquad Q(x) = -e^{3x}$$

$$\mu(x) = e^{\int p(x)} = e^{\int -1} = \boxed{e^{-x}}$$

$$v = \frac{1}{\mu(x)}\int \mu(x)\,Q(x)\,dx = \frac{1}{e^{-x}}\int e^{-x} \times -e^{3x}\,dx$$

$$v = -e^{x}\int e^{2x}\,dx = -e^{x}\times\frac{1}{2}e^{2x} \Rightarrow \boxed{v = \frac{-1}{2}e^{3x}}$$

$$u^{-1} = \frac{-1}{2}e^{3x} \Rightarrow (x+y) = \frac{-1}{2}e^{3x} \Rightarrow \boxed{y = \frac{-1}{2}e^{3x} - x}$$

#

السؤال العاشر :

if (dy/dx)=sin(t)*(y^2)*(e^cost) ,y(0)=4e^-1 find y(π/2)

$$\frac{dy}{dt} = \sin t\, y^2 e^{\cos t}, \quad y(0) = 4e^{-1} \quad \text{find } y(\tfrac{\pi}{2})?$$

Seperable.

$$\int\frac{dy}{y^2} = \int \sin t\, e^{\cos t}\,dt \Rightarrow \boxed{-y^{-1} = -e^{\cos t} + c}$$

$$\text{since } y(0) = 4e^{-1} \Rightarrow \frac{-1}{4e^{-1}} = -e^{\cos 0} + c \Rightarrow -4e = -e + c \Rightarrow \boxed{c = -3e}$$

$$y(\tfrac{\pi}{2}) \Rightarrow \frac{-1}{y} = -e^{\cos t} + -3e \Rightarrow \frac{-1}{y} = -1(e^{0} - 3e) \Rightarrow \frac{1}{y} = (1-3e) \Rightarrow \boxed{y(\tfrac{\pi}{2}) = \frac{1}{1-3e}}$$

السؤال الحادي عشر:

the equation y'+p(x)=sin^2(x) ,M(x)=sin(x) find p(x)

$y' + p(x)\,y = \sin^2(x) \quad , M(x) = \sin x \, , \quad \text{Find } P(x)$

$M(x) = e^{\int P(x)} \Rightarrow \sin x = e^{\int P(x)\,dx} \Rightarrow \ln \sin x = \int P(x)\,dx$

نشتق

$\frac{\cos x}{\sin x} = P(x) \Rightarrow \boxed{p(x) = \cot x}$

السؤال الثاني عشر :

xM(x,y)-yN(x,y)=0 on the folllowing is a solu
of M(x,y).dx+N(x,y).dy=0

$x M(x,y) - y N(x,y) = 0 \quad \text{Sol of} \quad M(x,y)\,dx + N(x,y)\,dy = 0$

$x M(x,y) = y N(x,y)$

$\frac{x}{y} M(x,y) = N(x,y)$

نعوض $\Rightarrow \left(M(x,y)\,dx + \frac{x}{y} M(x,y)\,dy = 0 \right) \div M(x,y)$

$1\,dx + \frac{x}{y}\,dy = 0 \Rightarrow \int \frac{1}{y}\,dy = \int -\frac{1}{x}\,dx \Rightarrow \ln y = -\ln x + c$

نرفع ل e

$\boxed{y = x^{-1} \cdot e^{c}}$

السؤال الثالث عشر:

if (N(x,y)-M(x,y))/(M(x,y))=y^2 then (M(x,y))/x.dx-(N(x,y)/y).dy=0 , find the D.E?

$$\frac{N(x,y)-M(x,y)}{M(x,y)} = y^2 \quad \text{then} \quad \frac{M(x,y)}{x}\,dx + \frac{N(x,y)}{y}\,dy = 0$$

Find the D.E ?

$$\rightarrow N(x,y) - M(x,y) = y^2 M(x,y) \Rightarrow N(x,y) = y^2 M(x,y) + M(x,y)$$

$$\boxed{N(x,y) = M(x,y)(y^2+1)}$$

نعوض هنا

$$\left(\frac{M(x,y)}{x}\,dx - \frac{M(x,y)(y^2+1)}{y}\,dy = 0\right) \div M(x,y)$$

$$\frac{1}{x}\,dx - y + \frac{1}{y}\,dy = 0$$

$$\int \frac{1}{x}\,dx = \ln x + g(y) \longrightarrow g'(y)$$

اشتقاق بالنسبة لـ y

$$\int g'(y) = \int -y + \frac{1}{y}\,dy \Rightarrow \boxed{g(y) = -\frac{y^2}{2} - \ln y}$$

بتكامل بالنسبة لـ y

$$\boxed{\ln x - \frac{y^2}{2} - \ln y = C} \Rightarrow \boxed{\ln x + C = \frac{y^2}{2} + \ln y}$$

#

السؤال الرابع عشر:

Find the equation (solve) :-2(1+x+xy^2)y'-y=y^3

$-2(1+x+xy^2)y' - y = y^3$

$-2(1+x+xy^2)y' = y^3+y \Rightarrow \dfrac{dy}{dx} = \dfrac{y^3+y}{-2(1+x+xy^2)}$

نقلب ⇐

$\dfrac{dx}{dy} = \dfrac{-2-2x-2xy^2}{y^3+y}$

$\dfrac{dx}{dy} = \dfrac{-2-2x(1+\cancel{y^2})}{y(\cancel{y^2}+1)} \Rightarrow \dfrac{dx}{dy} = \dfrac{-2(1+x)}{y}$

sepreable

$\int \dfrac{dx}{-2(1+x)} = \int \dfrac{dy}{y} \Rightarrow -\dfrac{1}{2}\ln|1+x| = \ln y$

ندخل e

$\boxed{(1+x)^{-\frac{1}{2}} = y}$ #

السؤال الخامس عشر:

(solve) (3y^2+2xy).dy-(2x-y^2).dx=0 ,y(1)=1 find y(0)

$\underbrace{(3y^2+2xy)}_{N(x,y)}dy - \underbrace{(2x-y^2)}_{M(x,y)}dx = 0$, $y(0)=?$, $y(1)=0$

$\boxed{M_y = 2y}$, $\boxed{N_x = 2y} \Rightarrow$ exact.

$\int -2x+y^2\,dx = -x^2+xy^2+g(y)$

اشتقه بالنسبة ل y

$2\cancel{x}y + g'(y) = 3y^2+2\cancel{x}y \Rightarrow \int g'(y) = \int 3y^2\,dy$

$\boxed{g(y) = y^3}$

$\boxed{y^3 - x^2 + xy^2 = c}$ $y(1)=1$ then $\Rightarrow 1-1+1=c \Rightarrow \boxed{c=1}$

$y^3 - x^2 + xy^2 = 1 \Rightarrow y(0) \Rightarrow y^3 - 0 + 0 = 1 \Rightarrow \boxed{y(0)=1}$ #

السؤال السادس عشر :

solve the I.P.V $y^2y'+\cos x.y^2=\cos x$,$y(\pi/2)=2$

$y^2y' + \cos x\, y^3 = \cos x$, $y(\frac{\pi}{2})=2$

$y' + \cos x\, y = \cos x\, y^{-2}$ Bernolli

$P(x) = \cos x$, $Q(x) = \cos x$ $n=-2$

$V = y^{(1-n)} \Rightarrow \boxed{V = y^3}$

$V' + (1-n)P(x)V = Q(x)(1-n)$

$V' + 3\cos x\, V = 3\cos x$ linear in V

$P(x) = 3\cos x$, $Q(x) = 3\cos x$

$\mu(x) = e^{\int P(x)} = e^{\int 3\cos x} = \boxed{e^{3\sin x}}$

$V = \frac{1}{\mu(x)}\int \mu(x)\, Q(x)\, dx = \frac{1}{e^{3\sin x}}\int e^{3\sin x}\cos x\, dx$

$V = \frac{1}{e^{3\sin x}}\left(e^{3\sin x}+c\right) \Rightarrow y^3 = e^{-3\sin x}\left(e^{3\sin x}+c\right)$

$\boxed{y(\frac{\pi}{2})=2} \Rightarrow 8 = e^{-3}(e^3+c) \Rightarrow 8 = 1 + ce^{-3} \Rightarrow 7 = ce^{-3}$

$\boxed{C = 7e^3}$ #

السؤال السابع عشر:

solve y.dx+x.dy+y^2(x.dy-y.dx)=0

Page 178 - Solve $y\,dx + x\,dy + y^2(x\,dy - y\,dx) = 0$

$y\,dx + x\,dy + xy^2\,dy - y^3\,dx = 0$

$(x + xy^2)\,dy + (y - y^3)\,dx = 0$

$(x + xy^2)\,dy = y^3 - y\,dx \Rightarrow \frac{dy}{dx} = \frac{(y^3 - y)}{x + xy^2}$

$\frac{dy}{dx} = \frac{y^3 - y}{x(1+y^2)} \Rightarrow \int \frac{1+y^2}{y^3 - y}\,dy = \int \frac{dx}{x}$

$u = 1 + y^2$

$du = 2y\,dy \Rightarrow \int \frac{u}{2y^2(y^2-1)}\,du = \int \frac{u}{2(u-1)(u-2)}\,du$ لـ تعويضنا

$y^2 = u - 1$

$\frac{A}{(u-1)} + \frac{B}{(u-2)} \Rightarrow A(u-2) + B(u-1) = u \Rightarrow \overset{u=1}{\boxed{A = -1}} \quad \overset{u=2}{\boxed{B = 2}}$

$\Rightarrow \frac{1}{2}\int \frac{-1}{u-1} + \frac{2}{u-2}\,du = \frac{1}{2}\left(-\ln|u-1| + 2\ln|u-2|\right)$

$\boxed{\frac{-1}{2}\{\ln|y^2| + \ln|y^2-1| = \ln|x| + c}$

السؤال الثامن عشر :

y'=x^3(y^2+x^2-2xy)+(2/x) ;substitution is u=y-x,solve it

السؤال التاسع عشر:

the general solution D.E xy'=y(lnx+lny)

$$xy' = y(\ln x + \ln y)$$

$$xy' = y \ln xy$$

$$V = xy \Rightarrow \boxed{V' = xy' + y} \qquad \boxed{y = \frac{v}{x}}$$

$$v' - y = y \ln v \Rightarrow v' = y \ln v + y \Rightarrow v' = y(\ln v + 1)$$

$$\frac{dv}{dx} = \frac{v}{x}(\ln v + 1) \Rightarrow \int \frac{dx}{x} = \int \frac{dv}{v(\ln v + 1)}$$

$$\ln|x| + c = \int \frac{1}{v(\ln v + 1)} dv$$

$$u = \ln v, \quad du = \frac{dv}{v} \Rightarrow \int \frac{\cancel{v}\, du}{\cancel{v}(u+1)} = \ln|u+1| = \ln|x| + c$$

$$\ln|\ln v + 1| = \ln|x| + c$$

$$= \boxed{\ln|\ln xy + 1| = \ln|x| + c}$$

السؤال العشرون :

find the general solution of the D.E:-
((5y^2)-xy+(y^3)*(siny))y'+xy^3=-y^2

السؤال الحادي و العشرون :

solve :- $(1+x^2).dy-x(3+3x^2-y)dx=0$

السؤال الثاني والعشرون:

solve :-$(\sin(xy)+xy\cos(xy)).dx+(1+(x^3)\cos(xy)).dy=0$

السؤال الثالث و العشرون:

if $M(x,y)=(1/(x^2+y^2)$ is an I.F of the D.E :-

$y.dx -(y^2+x^2+x).dy=0$, find the general solution

السؤال الرابع والعشرون :

solve :-$(xy).dy -(x^2+x\sqrt{(x^2+y^2)}).dx=0$

السؤال الخامس والعشرون:

an integrating factor :-$(3x^2).dx+(x^2y-x).dy=0$,is

السؤال السادس والعشرون :

the most general function $N(x,y)$that makes the following D.E exact is given by?? $N(x,y).dy+(\sin x\cos y-xy-e^{(xy)}).dx=0$

السؤال السابع و العشرون :

the general solution for the following exact D.E $(\sinh(x+y)+y^2e^x).dx+(\sinh(x+y)+2ye^x+1).dy=0$ is

السؤال الثامن والعشرون:

the ralation $x^2-\cos(x+y)=3$ defines an implicit solu of one of the following D.E , this D.E is

السؤال التاسع والعشرون :

find the general solution to the D.E

$(x+2y+1).dx+(3x+6y+2).dy=0$

السؤال الثلاثون:

solve the I.V.P (dy/dx)=√(x^2y)+(1/x)y , y(0)=1,x>0

$$\frac{dy}{dx} = \sqrt{x^2 y} + \frac{1}{x} y \Rightarrow \frac{dy}{dx} = x y^{\frac{1}{2}} + \frac{1}{x} y$$

$$\frac{dy}{dx} - \frac{1}{x} y = x y^{\frac{1}{2}} \Rightarrow (\text{Bernolli}) \quad n = 1/2 \quad P(x) = -1/x \quad Q(x) = x$$

$$u = y^{1-n} \Rightarrow u = y^{1-1/2} \Rightarrow u = y^{1/2}$$

$$u' = \frac{1}{2} y^{-1/2} \frac{dy}{dx}$$

$$\text{I.v.p} \Rightarrow u' + (1-n)\, p(x)\, u = (1-n)\, Q(x)$$

$$\text{I.v.p} \Rightarrow u' + \frac{-1}{2x} u = \frac{x}{2} \Rightarrow (\text{Linear in } u)$$

$$\mu(x) = e^{\int p(x)\cdot dx} \Rightarrow e^{\int \frac{-1}{2x} \cdot dx} \Rightarrow e^{-\frac{1}{2}\ln(2x)} = (2x)^{-\frac{1}{2}} = \frac{1}{\sqrt{2X}}$$

$$u = \frac{1}{\mu(x)} \left[c + \int \mu(x)\, Q(x) \cdot dx \right]$$

$$= \sqrt{2x} \left[c + \int \frac{1}{\sqrt{2x}} * \frac{x}{2} \cdot dx \right]$$

$$= \sqrt{2x}\left[c + \frac{1}{2\sqrt{2}} \int \frac{x}{\sqrt{x}} \cdot dx\right]$$

$$= \sqrt{2x}\left[c + \frac{1}{2\sqrt{2}} \int \sqrt{x} \cdot dx\right]$$

$$= \sqrt{2x}\left[c + \frac{\cancel{2}}{3} * \frac{1}{\cancel{2}\sqrt{2}} x^{\frac{3}{2}}\right] \Rightarrow \sqrt{2x}\left[c + \frac{1}{3\sqrt{2}} x^{3/2}\right]$$

السؤال الحادي والثلاثون:

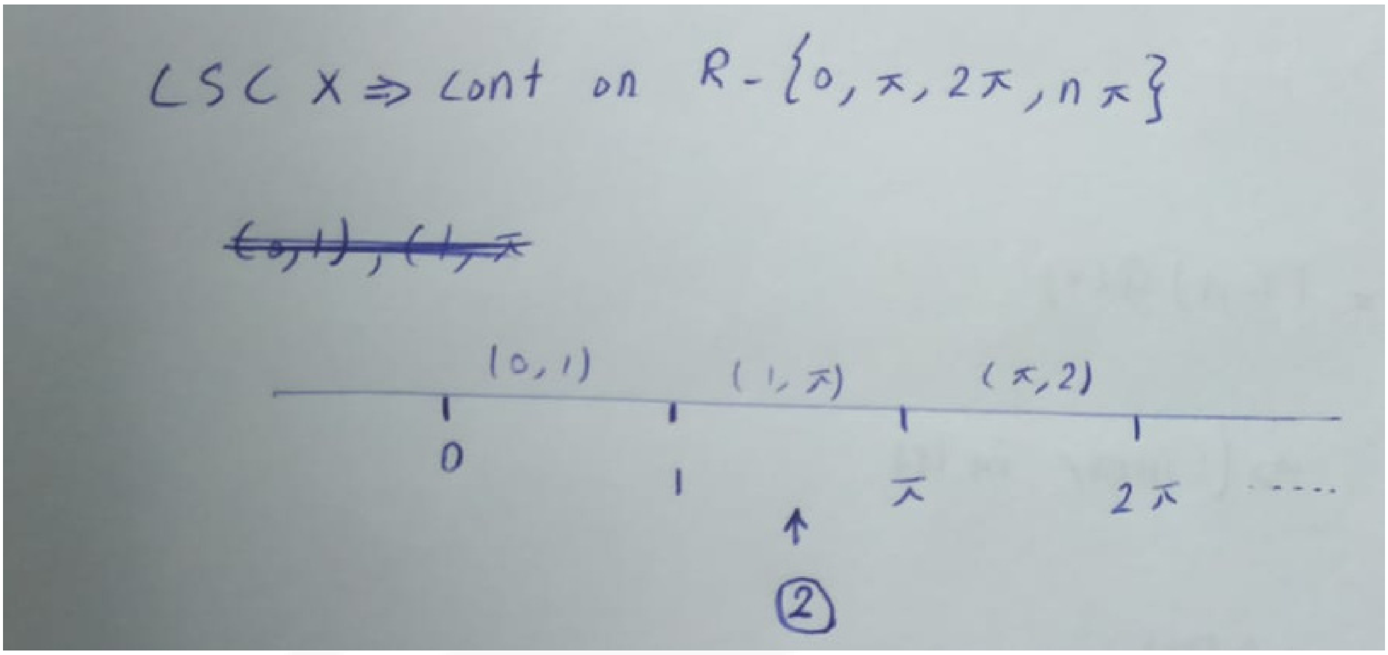
find the largest interval on which you are sure that the following initial value problem has a unique solution :-$\sqrt{\ }(x-1)y'+(\cos x)y=\csc x$,$y(2)=3$

$$\sqrt{x-1}\, y' + (\cos x)\, y = \csc x$$

$$y' + \frac{\cos(x)}{\sqrt{x-1}} y = \frac{\csc x}{\sqrt{x-1}}$$

$\cos(x) \Rightarrow$ cont on R

$\sqrt{x-1} \Rightarrow$ cont on $R - \{1\}$



السؤال الثاني والثلاثون :

consider the IVP :-y.dx+(y^2-x).dy=0 ,y(2)=1

a)find the integrating factor of the D.E

b) solve the initial value problem.

السؤال الثالث والثلاثون:

using a suitable substitution transform the D.E (cosy)y'+x(siny)=3 into a linear equation.

$$(\cos y)\, y' + x(\sin y) = 3$$

$$y' + x\,\frac{\sin y}{\cos y} = \frac{3}{\cos y}$$

$$y' + x(\tan y) = \frac{3}{\cos y}$$

$$\boxed{\begin{array}{l} \text{let } u = \sin y \\[1em] \dfrac{dy}{dx} = \cos y \; y' \end{array}}$$

$$\left(\frac{u'}{\cos y} + \frac{xu}{\cos y} = \frac{3}{\cos y}\right) * (\cos y)$$

$$u' + xu = 3 \Rightarrow \text{Linear in } u \;\#$$

السؤال الرابع والثلاثون:

consider the non-exact D.E :-
if (Py-Qx)/p=3 ,find an integrating factor to
p(x,y).dx+Q(x,y).dy=0

$$\frac{P_y - Q_x}{P} = 3 \Rightarrow \frac{Q_x - P_y}{P} = -3$$

$$\mu_{(y)} = e^{\int -3 \cdot dy} \Rightarrow \boxed{e^{-3y} = I.f.}$$

السؤال الخامس والثلاثون:

if you know (u=dy/dx) , find the value (A)in (u=3$\sqrt{\ }$y) ,use IVP,y(0)=0 ,y(3)=8

$$u = y' = A\sqrt[3]{y} \Rightarrow y' = A y^{1/3} \Rightarrow \frac{dy}{dx} = A y^{1/3} \Rightarrow (\text{sep})$$

$$\frac{dy}{y^{1/3}} = A\,dx \Rightarrow y^{-1/3} \cdot dy = A \cdot dx$$

$$\int y^{-1/3} \cdot dy = \int A \cdot dx \Rightarrow \frac{3}{2} y^{2/3} = A x$$

السؤال السادس والثلاثون:

solve :-$e^{(dy/dx)}+2e^{2xy}=x^2$

السؤال السابع والثلاثون:

solve :-$(1+2xy+x^2y^2).dx +x^2.dy=0$

السؤال الثامن والثلاثون:

if M(x,y)-N(x,y)/N(x,y)=x, then the sol to the D.E (M(x,y)/x).dx-(N(x,y)/y).dx=0 ,x,y>0

$$\frac{M(x,y)-N(x,y)}{N(x,y)} = x$$

$$M(x,y) - N(x,y) = x\,N(x,y)$$

$$M(x,y) - N(x,y) - x\,N(x,y) = 0$$

$$M(x,y) - N(x,y)(1+x) = 0 \Rightarrow M(x,y) = N(x,y)(1+x)$$

$$\frac{M(x,y)}{N(x,y)} = 1+x$$

$$\frac{M(x,y)}{x}\cdot dx - \frac{N(x,y)}{y}\cdot dy = 0 \qquad \div N(x,y)$$

$$\frac{M(x,y)}{N(x,y)} * \frac{1}{x} - \frac{1}{y} = 0 \Rightarrow \frac{M(x,y)}{N(x,y)} = \frac{x}{y}$$

$$\frac{x}{y} = 1 + x \qquad \frac{M(x,y)}{N(x,y)} = 1 + x$$

$$\frac{1}{y} \cdot dy = \frac{1+x}{x} \cdot dx \Rightarrow \int \frac{1}{y} \cdot dy = \int \frac{1+x}{x} \cdot dx$$

$$\boxed{\ln|x| + x + C = \ln|y|} \; ※$$

السؤال التاسع والثلاثون :

the values of m,n that make the D.E

(2√ x(x-2y)^m).dx-(x^3-2n-5).dy=0 homogeneous are

$$2x^{1/2}(x-2y)^{m} \cdot dx = (x^3 - 2n - 5) \cdot dy$$

$$\frac{1}{2} + m = 3$$

$$m = \frac{5}{2}$$

* ملاحظة :- في حالة الـ homog يجب أن تتساوى الأسس .

* ملاحظة :- يجب ألا تحتوي على ارقام (ثوابت) فقط متغيرات .

$$-2n - 5 = 0$$

$$-2n = 5$$

$$n = \frac{-5}{2}$$

السؤال الاربعون :

the solution of the D.E y'=x^2+2xy+y^2-1

$$y' = \underbrace{x^2 + 2xy + y^2}_{\text{عبارة تربيعية}} - 1$$

$$y' = (x+y)^2 - 1$$

$$v = x + y \Rightarrow v' = 1 + y' \Rightarrow y' = \cancel{1} + v' - 1$$

(sep)

$$\Downarrow$$

$$y' = (x+y)^2 - 1 \Rightarrow v' - \cancel{1} = v^2 - \cancel{1} \Rightarrow \frac{dv}{dx} = v^2 \Rightarrow \frac{dv}{v^2} = dx$$

$$\int \frac{dv}{v^2} = \int dx \Rightarrow \int v^{-2}\,dv = \int dx \Rightarrow \boxed{\frac{-1}{v} = x + c} \;\divideontimes$$

$$\boxed{\frac{-1}{x+y} = x + c} \;\divideontimes$$

السؤال الحادي والاربعون :

for what the value of (k) is (x^2+y^2)^k an integrating factor for -y.dx+x.dy=0

$-y\,dx + x\,dy = 0 \Rightarrow$ (non-exact)

by $*\,(x^2+y^2)^K$

$-(x^2+y^2)^K y\,dx + x(x^2+y^2)^K\,dy = 0 \Rightarrow$ (exact)

$M_y = -K(x^2+y^2)^{K-1} * 2y * y - (x^2+y^2)^K \Rightarrow -K(x^2+y^2)^{K-1} * 2y^2 - (x^2+y^2)^K$

$N_x = K(x^2+y^2)^{K-1} * 2x * x + (x^2+y^2)^K \Rightarrow 2Kx^2(x^2+y^2)^{K-1} + (x^2+y^2)^K$

$\underline{M_y = N_x}$

$-2Ky^2(x^2+y^2)^{K-1} - (x^2+y^2) = 2Kx^2(x^2+y^2)^{K-1} + (x^2+y^2)^K$

$2Kx^2(x^2+y^2)^{K-1} + 2Ky^2(x^2+y^2)^{K-1} + 2(x^2+y^2)^K = 0 \quad \div 2$

$Kx^2(x^2+y^2)^{K-1} + Ky^2(x^2+y^2)^{K-1} + (x^2+y^2)^K = 0$

$$(x^2+y^2)^K \left(1 + \frac{Kx^2+Ky^2}{x^2+y^2}\right) = 0$$

$$1 + K\left(\frac{\cancel{x^2+y^2}}{\cancel{x^2+y^2}}\right) = 0 \Rightarrow \boxed{K=-1}\ \#$$

السؤال الثاني والاربعون:

show that if y1,y2 are solution of the linear equation of order (n) , L[y]=0 ,then c1y1+c2y2 ,where c1,c2£R ,is also of L[y]=0

$$\left.\begin{aligned} L[y_1] &= a_1(x)\, y' + a_2(x)\, y'' + \cdots + a_n(x)\, y^{(n)} = 0 \\ L[y_2] &= a_1(x)\, y' + a_2(x)\, y'' + \cdots + a_n(x)\, y^{(n)} = 0 \end{aligned}\right\} \rightarrow$$

من تعريف Linear homo $\quad L[y] = a_1(x)\, y' + a_n(x)\, y^{(n)}$

so :

$$L[c_1 y_1 + c_2 y_2]$$

$$= a_n(x)\,(c_1 y_1 + c_2 y_2)^{(n)} + a_{n-1}(x)\,(c_1 y_1 + c_2 y_2)^{(n-1)} + \cdots + a_1(x)\,(c_1 y_1 + c_2 y_2)'$$

$$= a_n(x)\left(c_1 y_1^{(n)} + c_2 y_2^{(n)}\right) + a_{n-1}(x)\left(c_1 y_1^{(n-1)} + c_2 y_1^{(n-1)}\right) + \cdots + a_1(x)\left(c_1 y_1' + c_2 y_2'\right)$$

$$= c_1 a_n(x)\, y_1^{(n)} + c_1 a_{n-1}(x)\, y_1^{(n-1)} + \cdots + c_1 a_1(x)\, y' + c_2 a_n(x)\, y_2^{(n)} + c_2 a_{n-1}(x)\, y^{(n-1)} + \cdots + c_2 a_1(x)\, y_2'$$

$$= c_1 \underbrace{\left[a_n(x)\, y_1^{(n)} + a_{n-1}(x)\, y_1^{(n-1)} + \cdots + a_1(x)\, y'\right]}_{y_1} + c_2 \underbrace{\left[a_n(x)\, y_2^{(n)} + a_{n-1}(x)\, y^{(n-1)} + \cdots + a_1(x)\, y'\right]}_{y_2}$$

$$= c_1 [L y_1] + c_2 [L y_2]$$

Note:- $L[y_1] = L[y_2] = 0$ لانهم حلول .

$$= c_1 [0] + c_2 [0]$$

السؤال الثالث والاربعون :

find the general solution to the D.E :-

-2(1+x+xy^2)y'-y=y^3

$$y' = \frac{y^3+y}{-2(1+x+xy^2)} \Rightarrow \frac{dy}{dx} = \frac{y^3+y}{-2(1+x+xy^2)}$$

$$\frac{dy}{dx} = \frac{y^3+y}{-2-2(x+xy^2)} \Rightarrow \frac{dy}{dx} = \frac{y(1+y^2)}{-2-2x(1+y^2)} \leftarrow$$

نقلبها متى نوزع المقام.

$$\frac{dx}{dy} = \frac{-2-2x(1+y^2)}{y(1+y^2)} \Rightarrow \frac{dx}{dy} = \frac{-2}{y(1+y^2)} - \frac{2x\cancel{(1+y^2)}}{y\cancel{(1+y^2)}}$$

$$\frac{dx}{dy} = \frac{-2}{y(1+y^2)} - \frac{2x}{y} \Rightarrow x' + \frac{2x}{y} = \frac{-2}{y(y^2+1)}$$ (linear in x)

$$\mu(y) = e^{\int \frac{2}{y} \cdot dy} = e^{2\ln y} = y^2$$

$$x = y^{-2}\left[c + \int y^{\cancel{2}} * \frac{-2}{\cancel{y}(1+y^2)} \cdot dy\right]$$

$$x = y^{-2}\left[c + \int \frac{-2y}{1+y^2} \cdot dy\right]$$

* let $u = 1+y^2$

$du = 2y \cdot dy$

$\frac{du}{2y} = dy$

$$x = y^{-2}\left[c + \int \frac{-2\cancel{y}}{u} \cdot \frac{du}{2\cancel{y}}\right]$$

$$x = y^{-2}\left[c + \int \frac{-1}{u} \cdot du\right]$$

$$x = y^{-2}\left[c + -\ln(x)\right]$$

$$x = y^{-2}\left[c + -\ln(1+y^2)\right] \divideontimes$$

السؤال الرابع والاربعون :

find the largest interval for which the IVP :

y'+(y/xln(e^x-3))=x , y(ln(3/2))=4 has a unique

السؤال الخامس والاربعون:

consider the D.E :M(x,y).dx+N(x,y).dy=0

if (My)^2-(Nx)^2=cos(x)(My+Nx)N(x,y) and cosx>0, (My+Nx)≠0 find the integrating factor of the D.E ??

$$(M_y - N_x)\cancel{(M_y + N_x)} = \cos(x)\cancel{(M_y + N_x)}\,N(x,y)$$

$$(M_y - N_x) = \cos(x)\; N(x,y)$$

$$\frac{(M_y - N_x)}{N(x,y)} = \cos(x) \leftarrow \text{only in } \underline{\underline{x}}$$

$$\mu(x) = e^{\int \frac{M_y - N_x}{N(x,y)}\,dx} = e^{\int \cos x} = e^{\sin x} = I.F$$

السؤال السادس والاربعون :

the suitable substitution that transform theD.E :- ye^(xy)(dy/dx)+xe^(xy)=12y^2 ,x,y>0 , into a seprable equation is ??

let $u = e^{xy} \Rightarrow \ln u = xy \Rightarrow x = \frac{\ln u}{y}$

$$x' = \frac{y\frac{u'}{u} - \ln u}{y^2}$$

$$y.u.x' + \frac{\ln u}{y}.u = 12y^2 \Rightarrow \cancel{y}\cancel{u}\left(\frac{yu' - u\ln u}{\cancel{u}.y^{\cancel{2}}}\right) + \frac{\ln u}{y}.u = 12y^2$$

$$= \frac{y.u' - u\ln u}{y} + \frac{\ln u.u}{y} = 12y^2$$

$$\frac{\cancel{y}.u'}{\cancel{y}}\left(-\cancel{\frac{u\ln u}{y}}\right)\left(+\cancel{\frac{u\ln u}{y}}\right) = 12y^2 \Rightarrow u' = 12y^2$$

$$\frac{du}{dy} = 12y^2 \Rightarrow \int 1\,du = \int 12y^2.dy \Rightarrow u = 4y^3 + c \quad ※$$

(sep)

السؤال السابع والاربعون:

(M(x,y)-N(x,y))/(N(x,y))=x , then the solution to the D.E :-M(x,y)/x).dx-N(x,y).dy=0 ,,x,y>0 is??

$$\frac{M(x,y)}{x}.dx = \frac{N(x,y)}{y}.dy \Rightarrow \frac{M(x,y)}{N(x,y)} = \frac{x}{y}.\frac{dy}{dx}$$

$$\frac{M(x,y)-N(x,y)}{N(x,y)} = x \Rightarrow \frac{M(x,y)}{N(x,y)} - 1 = x \Rightarrow \frac{M(x,y)}{N(x,y)} = x+1$$

(sep)

$$\frac{x}{y}\frac{dy}{dx} = x+1 \Rightarrow \frac{1}{y}.dy = 1+\frac{1}{x}.dx \Rightarrow \ln y = x + \ln x + c$$

السؤال الثامن والاربعون :

classify each of the following equation as to :- separable, homo ,exact ,linear and bernoulli ?

a)(dy/dx)=(xy^2+x-y^2-1)/(yx^2+2y-3x^2-6).

$$\frac{dy}{dx} = \frac{(xy^2+x)-(y^2+1)}{(yx^2+2y)-3(x^2+2)} \Rightarrow \frac{dy}{dx} = \frac{x(y^2+1)-(y^2+1)}{y(x^2+2)-3(x^2+2)}$$

نسحب (y^2+1) عامل مشترك من البسط

$$\frac{dy}{dx} = \frac{(y^2+1)(x-1)}{(y-3)(x^2+2)}$$

ونسحب (x^2+2) عامل مشترك من المقام

$$\frac{(y-3).dy}{(y^2+1)} = \frac{(x-1)}{(x^2+2)}.dx \quad (sep)$$

b)y'=1/(x(x-y))

$$y' = \frac{1}{x(x-y)} \Rightarrow \frac{dy}{dx} = \frac{1}{x(x-y)} \Rightarrow \frac{dx}{dy} = x(x-y)$$

$$x' = x^2 - xy \Rightarrow x' + xy = x^2 \quad (\text{Bernolli})$$

c) y'-5y=4+y^2

$$y' - 5y = 4 + y^2$$

$$y' = y^2 + 5y + 4 \Rightarrow y' = (y+4)(y+1) \Rightarrow \frac{dy}{dx} = (y+4)(y+1)$$

$$\frac{dy}{(y+4)(y+1)} = 1 \cdot dx \quad (\text{sep})$$

d)(1+x^2).dy+(xy+x^3+x).dx=0

$$(1+x^2) \cdot dy = -(xy + x^3 + x) \cdot dx$$

$$\frac{dy}{dx} = \frac{-(xy + x^3 + x)}{(1+x^2)} \Rightarrow y' = \frac{-xy}{1+x^2} - \frac{x\cancel{(x^2+1)}}{\cancel{(x^2+1)}}$$

$$y' + \frac{xy}{1+x^2} = -x \quad (\text{linear}) \; ※$$

السؤال التاسع والاربعون :

solve $y'=(2x+\cos y)/(x\cos y+2y+1)$

السؤال الخمسون :

find the integrating factor of the differential equaltion $(yx^3+2x^4-4y).dx+(x^4+2x).dy=0$

السؤال الحادي والخمسون :

using a suitable substitution to transform the D.E:- $(2x-2).dy-(2x+y).dx=-y.dy$ into a separable equ the resulting equation is given by:

السؤال الثاني والخمسون :

let y(x) the solu for the D.E $(y-xe^x).dx+(x+2).dy=0$; $x>-2$ with $y(1)=3$,find $y(0)$??

السؤال الثالث والخمسون :

the best substitution to transform the D.E:-
(x^3-yx^2)y'+x=0 into linear equ is given by

السؤال الرابع والخمسون:

let y(x) be the solu for the D.E:-
y.dx=(xy-2y^2).dy=0 ,if y(1/e)=1 ,then at x=-2 the value of y wil satisfy ??

السؤال الخامس والخمسون :

when using the substitution u=yx in the D.E
(x^3y^2+x^2y+x).dy+y(1+xy).dx=0 the resulting differ equa will be

السؤال السادس والخمسون :

using thesuitable substitution to transformthe D.E :-
y.dx+(x+$\sqrt{}$(xy)).dy=0 into separable equation the resulting D.E will be

السؤال السابع والخمسون :

find the value of K such that the substitution $u=y/(x^k)$ transform the D.E:-
$(Y^2+3X^4).dx+(5xy.dy=0$ into a separable equa

السؤال الثامن والخمسون:

find the general solu to the D.E :-
$(\sin 2x+2\ln y).dx+((2x/y)+ye^y).dy=0$

السؤال التاسع والخمسون :

solve $(xy^2-y^2-x+1)\,y'+y+x+xy=1 \quad ,y\neq-1$

السؤال الستون:

find a suitable substitution to transform the D.E:-
$(y'/(1+4y^2))+((x^2-\tan^{-1}(2y))/(x+1))=1/y$ into a linear equa ,then find the resulting linear equa.
(don't solve the equa)

السؤال الحادي والستون :

if $M(x)=e^{h(x)}$ is integrating factor of the linear D.E:-
$x(dy/dx)-h(x)=\sin(x)-x^2y$ then the function h(x) is given by

السؤال الثاني و الستون :

$(ye^{(x/y)}).dx-(xe^{(x/y)}-3y^2).dy=0$, $y\neq0$,is it possible to solve the equa using the substitution $u=x/y$, if yes find the general solu , if no find another method.

السؤال الثالث والستون :

find the values of K and B that make the D.(E :-
$(1/x^2+2)+(K/y)).dx+(xy^{(3B)}+1).dy=0$ an exact

السؤال الرابع والستون :

solve (((siny)/y)-2e^).dx+((cosy+2e^-2xcosx)/y).dy=0

Hint :try an I.F of the form M(x,y)=ye^x

السؤال الخامس والستون :

solve the following I.V.P:-

a) y^2y'+costy^3=cost ,y(π/2)=2

b)xyy'=x^e^(y/x)+y^2 ,y(1)=2

السؤال السادس والستون :

find the general solu for:

e^(y)y'-(e^-x)/(x^2+xy)+e^y=0

السؤال السابع والستون:

show that the differ equa y'=f(x,y) is homo iff f(tx,ty)=f(x,y)

السؤال الثامن والستون :

show that the if $(Nx-My)/(xM-yN)=Q$, where Q depend only on the quantity xy ,then the differ equa:- $M+Ny'=0$, has an I.F of the form $M(x,y)$
give the general formula for this integrating factor ??

السؤال التاسع والستون :

if y(x) is solu to the D.E :-
$xy'+(2x+1)y-e^{-2x}=0$,with $y(1)=e^{-2}$ find value of y(2)

السؤال السبعون :

find the value of B so that the substitution $y=vx^B$ transform the D.E $(x+3x^2y)y'+xy^2=y$ into a separable

السؤال الحادي والسبعون:

find the general solu to the D.E:-

(y^2e^(xy^2)).dx+(2xye^(xy^2)).dy=0

السؤال الثاني والسبعون :

find the general solu to the D.E:-

y'+2x-2(x^2+y-1)^(3/2)=0

السؤال الثالث والسبعون :

if M(x,y)=1/(x^2-y^2) is an I.F of the D.E :-

(x^2-y^2-y)-(x^2-y^2-x)y'=0 . then the general solu is given by

السؤال الرابع والسبعون :

find the integrating factor of the D.E :-

e^x(x+1)+(ye^y-xe^x)y'=0 ,then find general solu??

السؤال الخامس والسبعون :

using the suitable u=xy for the D.E :-
$x(xy-1)^2y'+(x^2y^2+1)y=0$

السؤال السادس والسبعون :

find a substitution to transform the D.E:-
$(x^2-1)y'+xy-3xy^2=0$ into a linear equa then find the result linear equa (don't solve equa).

السؤال السابع والسبعون :

find the value of the D.E :-
$dy=((1/x)-(1/y^2+1)-(1/x(y^2+1))+1).dx$

السؤال الثامن والسبعون :

find the general solu of the D.E:-
$x(2x-y^3)y'-y^4=2xy$

TURBO